Influence of Optical Dispersion on the Performances of an Electrooptic Phase Modulator

P. Chazan, M. Haelterman, and S. Tedjini

Abstract—**An equation for electrooptic modulation taking the optical dispersion into account is established using the standard perturbation method. The influence of the optical dispersion on the characteristics of an electrooptic phase modulator is analyzed with a numerical approach. The existence of an amplitude modulation due to the optical dispersion is demonstrated. For high-modulation frequencies, the dispersion can affect significantly the response of the phase modulator. The modulation factor can reach 50%.**

I. Introduction

Ultra high-speed electrooptic modulators are important devices that have a wide range of potential applications especially in optical telecommunications. Today, several gigabit systems are in use and several ten gigabit systems are under development. Modulation capabilities in excess of 100 GHz have been demonstrated [1]. These devices are based on the phase modulation induced by Pockels' effect. It is well established that the bandwidth of these components is mainly limited by the mismatch between the group velocity of optical signal and the phase velocity of the microwave signal [2], [3]. Usually, the group velocity of the optical signal is considered to be constant. This is a good approximation for low-frequency electrooptic modulation. However, for millimeter-wave and submillimeter-wave domains, the dispersion of the group velocity must be taken into account.

For the first time, we study numerically the effect of the frequency dependence of the optical group velocity on electrooptic phase modulation. Physically, the dispersion will generate an amplitude modulation, in addition to the phase modulation. In fact, in a phase modulation, the optical carrier and the two sidebands are in quadrature. During the propagation, the phase difference between the carrier and the sidebands changes due to dispersion. There is now some in-phase component and this is the basis of amplitude modulation. In this letter, the importance of this amplitude modulation is studied, as well as its effect on the phase modulation. It is shown that this phenomenon may strongly affect the performances of high-frequency electrooptic phase modulators.

II. Theory

In order to take the dispersion of the group velocity into account for the propagation of the optical wave, we have used a perturbation method. We describe here the main steps of the calculation.

We begin with the scalar, nonlinear-wave equation [4]:

$$\nabla^2 E - \frac{\varepsilon_r}{c^2}\frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_{\rm NL}}{\partial t^2}, \quad (1)$$

where ε_r is the relative permittivity of the optical guide, E is the optical field, $P_{\rm NL}$ is the nonlinear polarization induced by the microwave field via the second-order susceptibility (Pockels' effect).

For particular modulation conditions in AsGa and $LiNbO_3$ $P_{\rm NL}$ can be expressed as [5]:

$$P_{\rm NL}(r,t) = \varepsilon_0 n^4 r_e E_m \varphi_m(x,y)\cos(\omega_m t - \beta_m z)E(r,t), \quad (2)$$

where $r = x,y,z$; z is the direction of propagation, r_e is the electrooptic coefficient corresponding to the direction of the microwave field. $\varphi_m(x,y)$, n, E_m, ω_m, β_m are the normalized mode profile, the microwave index, the maximum amplitude, the angular frequency and the propagation constant of the microwave mode, respectively.

We assume the following form for the optical field:

$$E(r,t) = \int C^\omega(z)\varphi(x,y)e^{i(\beta(\omega)z-\omega t)}d\omega, \quad (3)$$

where $\varphi(x,y)$ is the normalized mode profile of the monomode optical guide. $\beta(\omega)$ is the propagation constant, C^ω is the amplitude of the spectral component at angular frequency ω. In the wave equation, the nonlinear polarization leads to a coupling of the spectral components of the optical mode. Thus, their amplitude C^ω change during the propagation and it is a function of z.

Substituting (3) into (1), using the slowly varying amplitude approximation (SVA) and using the normalization of the mode, we get:

$$\int 2i\beta(\omega)\frac{\partial C}{\partial z}e^{i(\beta(\omega)z-\omega t)}d\omega = \int_s \varphi^*(x,y)\mu_0\frac{\partial^2 P_{\rm NL}}{\partial t^2}\,dxdy, \quad (4)$$

where S is the optical guide cross section.

Defining the slowly varying envelope:

$$\phi(z,t) = \int C^\omega(z)e^{i((\beta-\beta_0)z-(\omega-\omega_0)t)}d\omega, \quad (5)$$

where ω_0 is the central frequency of the optical spectrum.

Using an expansion of β to the second-order

$$\beta(\omega) = \beta(\omega_0) + \beta_0'(\omega-\omega_0) + \beta_0''\frac{(\omega-\omega_0)^2}{2}, \quad (6)$$

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where β_0'' s^2/m is defined as a "dispersion factor." After some algebraic transformations and neglecting the third-order terms, we get:

$$\frac{\partial\phi}{\partial z}+\beta_0'\frac{\partial\phi}{\partial t}+i\frac{\beta_0''}{2}\frac{\partial^2\phi}{\partial t^2} = \frac{\Gamma}{2i\beta_0}\varepsilon_0\mu_0 n^4 r\omega_0^2 E_m\cos(\omega_m t-\beta_m z)\Phi(z,t), \quad (7)$$

where Γ is the classical overlap integral between the applied microwave field and the optical mode.

We now define the new variables,

$$\zeta = z, \tau = t-\beta_0' z \quad (8a)$$

and the coefficients,

$$\alpha = \omega_m\beta_0' - \beta_m, K = \frac{n^4 r\omega_0^2}{2c^2\beta_0}\Gamma E_m(\text{rad/m}). \quad (8b)$$

Equation (7) can thus be rewritten in the form

$$\frac{\partial\phi}{\partial\zeta}+i\frac{\beta_0''}{2}\frac{\partial^2\phi}{\partial\tau^2} = iK\cos(\omega_m\tau+\alpha\zeta)\phi(\zeta,\tau). \quad (9)$$

α is the mismatching between the microwave and optical fields and K represents the coupling efficiency between the two fields.

If we solve (9), neglecting the second-order term, we find the classical expression for the phase modulation [6].

III. Numerical Results

The aim of this section is to analyze the effect of group velocity dispersion on the modulated wave. Special attention will be paid to the dispersion induced amplitude modulation that cannot be neglected in some cases of practical interest. Equation (9) has been solved in the spectral domain using a finite difference method

Our program has been checked by solving (9) in the case of zero dispersion and by comparison with the analytical solution. The numerical error was typically less than 0.004%.

The first step in analyzing the influence of the dispersion factor is to determine a good approximation of β_0'' in the considered material. For bulk gallium arsenide, we have calculated β_0'' from E. D. Palik's data [7]; its average value is $2.10^{-24}s^2/m$ at 1.3-μm wavelength. The guiding dispersion is generally much less than the dispersion in bulk material. Consequently, the optical guiding does not affect the values of β_0'' given here.

Initially, we have looked for a phase distortion due to the optical dispersion. For low values of the dispersion factor ($<10^{-23}s^2/m$), the phase modulation is not affected by the dispersion and the amplitude modulation is rather small, as shown in Fig. 1. For higher values of the dispersion factor, $10^{-22}s^2/m$ and $<10^{-21}s^2/m$, the phase distortion is important and the modulation factor goes up to 20% (this is not shown on Fig. 1). As far as we know, there is no electrooptic material having such high values of dispersion.

In order to characterize this amplitude modulation, we define the modulation factor:

$$\delta = \frac{\text{peak to peak amplitude of } |\phi(z,t)|}{\text{value of } |\phi(z,t)| \text{ if } \beta_0''=0}. \quad (10)$$

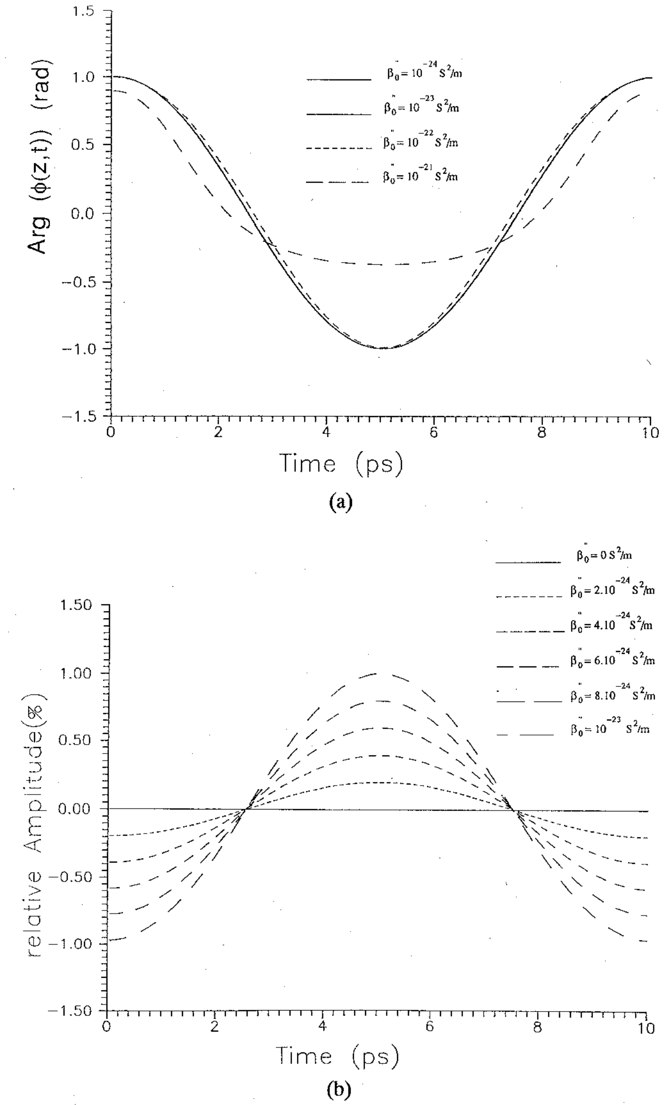

Fig. 1 Influence of the dispersion factor K=100, f=100 GHz. α=0. (a) Time evolution of the phase of the low-frequency envelope. (b) Time evolution of the magnitude of the low-frequency envelope.

We will now concentrate the analysis on dispersion values lower than $5.10^{-23}s^2/m$. In this dispersion domain, the modulation factor is proportional to the dispersion factor as shown in Fig. 2. With a modulating field of 1 V/μm, a typical value of K is 100 rad/m in gallium arsenide and 720 rad/m in lithium niobate. For a mismatching, α=100 and for a dispersion factor β_0''=$2.10^{-24}s^2/m$, the modulation factor is no more than 0.04% in GaAs. Although this amplitude modulation is negligible, it may be experimentally observed.

For materials with a dispersion coefficient β_0''=$10^{-23}s^2/m$, the modulation factor is 2% at 100 GHz and might be up to 20% if the value of K is increased as shown in Fig. 3(a). The case β_0''=$10^{-23}s^2/m$, corresponds to GaAs for an optical signal of 1-μm wavelength. It is important to point out that relatively high values of K (>500) considered in Fig. 3(a) are easily achievable in $LiNbO_3$ for normal modulation conditions, or for GaAs with modulating fields of 10 V/μm.

Now, if we suppose perfect microwave and optical matching (α=0, K= 100 rad/m), we may increase the microwave

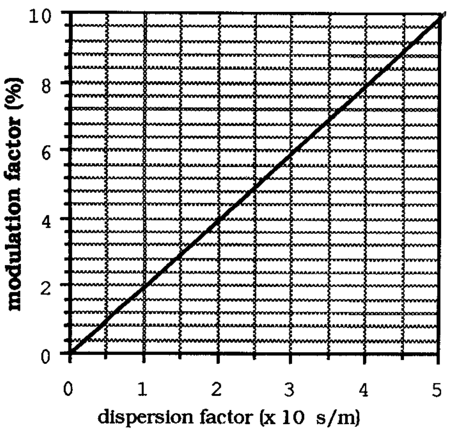

Fig. 2 α=0, K=100, f=100 GHz. Modulation factor for low values of the dispersion factor.

frequency. For the frequency range 1–100 GHz, the amplitude modulation does not exceed 2% for $\beta_0''=2.10^{-24}s^2/m$ and 5% for $\beta_0''=10^{-23}s^2/m$. However, for the frequency range 100–600 GHz, the modulation factor may be more than 10% for $\beta_0''=2.10^{-24}s^2/m$ and more than 50% for $\beta_0''=10^{-23}s^2/m$ (Fig. 3(b)). The electrooptic device is then a phase and an amplitude modulator. Consequently, for modulation frequencies exceeding 200 GHz, pure phase modulation seems to be difficult. Here, the dispersion of the material appears to limit the use of electrooptic effect for phase modulation.

IV. Conclusion

We have developed a phase modulation equation taking into account the chromatic material dispersion. The dispersion induces a change in the phase modulation spectrum that is equivalent to a slight amplitude modulation. For common devices this modulation may be neglected, but it is important enough to be viewed. For particular devices working at very high frequencies, or using highly dispersive electrooptic materials ($\beta_0'' > 10^{-23}s^2/m$), this amplitude modulation interferes with the phase modulation. Under this condition, the effect is far from negligible and should be taken into account.

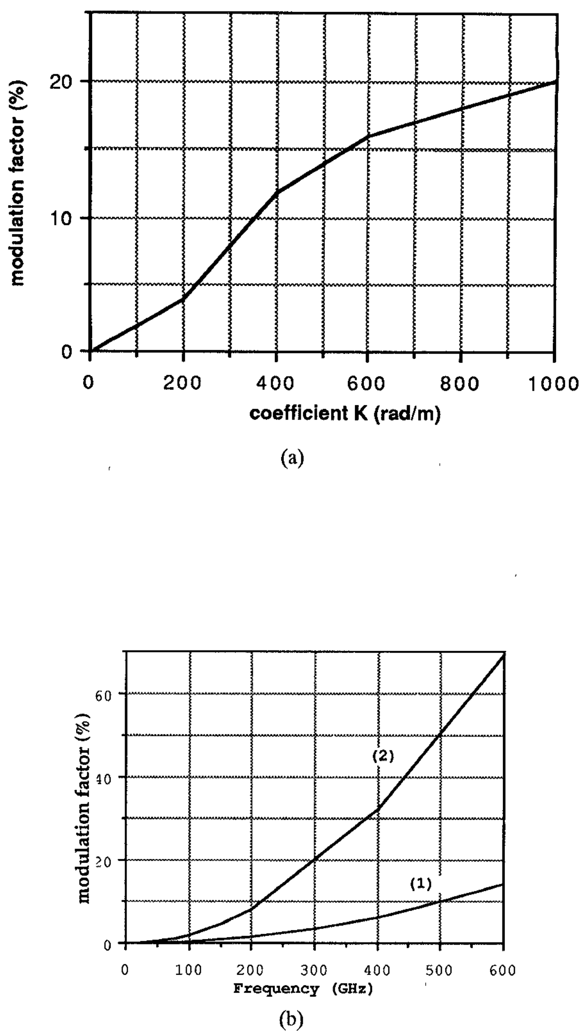

Fig. 3 α=0. Evolution of the modulation factor. (a) For different values of coefficient $K(f=$ 100 GHz, $\beta_0''=1-^{-23}s^2/m)$. (b) For different values of the frequency (K=100). $\beta_0''=2.10^{-24}s^2/m$, case (1). $\beta_0''=10^{-23}s^2/m$, case (2).

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